12. G. Astarita, G. Marrucci, and G. Palumbo, "Non-Newtonian gravity flow along inclined plane surface," Ind. Eng. Sci. Fund., 3, No. 4 (1964).
13. P. J. Carreau, Q. H. Bui, and P. Leroux, "Wall effects in polymer flow on an inclined plane," Rheol. Acta, 18, No. 5 (1979).
14. A. Dutta and R. A. Mashelkar, "On slip effect in free coating of non-Newtonian fluids," Rheol. Acta, 21, No. 1 (1982).

PRESSURE FLOW OF LIQUID WHICH CONGEALS ON A PIPE SURFACE UNDER CONDITIONS OF DISSIPATIVE HEAT RELEASE

S. V. Maklakov, A. M. Stolin, and S. I. Khudyaev

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There are many known processes in nature and in engineering where the flow of liquid is accompanied by a phase transformation. Examples of such processes are the accidental overcooling of pipelines [1], the transport of highly paraffinous petroleum [2], the motion of magma along a dike [3], the high-velocity flow of gas past an object [4], or even the electrical heating of a conductor during phase transformation [5]. The substantial effect of volumetric heat release during phase transition is shown in [4, 5]. An important peculiarity in these examples is the simultaneous interaction of the phase transition with chemical, Joule, or dissipative heat release. Earlier consideration has been made of the effect of the phase transition on the critical conditions of thermal shock in planar [6] and in cylindrical [7] regions and of its hydrodynamic analog, Couette flow [8].

This study investigates the peculiarities of the phase transition under conditions of viscous liquid pressure flow inside a pipe of circular cross section and of infinite length where there exists a given pressure gradient and a given flow rate. Either a constant temperature or a constant thermal flow is applied to the wall of the pipe.

It was shown that in all ranges of the parameters for the problem, a steady-state solution is achieved. Steady-state temperature and velocity profiles are determined. For a given pressure gradient and wall temperature in the quasisteady-state approximation, a plot is given for ranges of the parameters corresponding to the characteristic type of flow: a steady-state condition with intermediate positioning of the phase boundary, the condition of pipe capping (total phase transformation), and the condition of hydrodynamic thermal shock [9]. It was shown that for a given thermal flow on the wall the condition for intermediate positioning of the phase boundary is absent.

The peculiarities of flow for a given flow rate are analyzed. In this case, the steady-state flow with intermediate positioning of the phase boundary always exists. For a given thermal flow on the wall of the pipe it is possible to have flow without the solid phase. The flow rate and pressure characteristics are obtained, and the effect of phase transition is discussed.

1. Statement of the Problem. We will consider a phase transition of the first kind under the conditions of viscous, Newtonian liquid pressure flow inside a pipe of circular cross section and of infinite length whose walls are maintained at a constant temperature T_0 which is less than the temperature of phase T_*. Because of cooling, the liquid solidifies and a phase division is created on the inner surface at $r = r_*$. The dependence of viscosity on temperature goes according to the law of Arrhenius: $\eta = \eta_0 \exp(E/RT)$, where η_0 is a pre-exponential factor, E is the initiation energy of viscous flow, R is the universal gas constant, and T is temperature.

The equations of motion and heat balance, taking into account dissipative heat release, and the boundary conditions, can be written in the form

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$$r<r_*:\frac{1}{a_1}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\tau_*}{\lambda_1}\frac{\partial v}{\partial r}; \quad (1.1)$$

$$\rho_1\frac{\partial v}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}(r\tau_*)+b; \quad (1.2)$$

$$r>r_*:\frac{1}{a_2}\frac{\partial T}{\partial t}=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}; \quad (1.3)$$

$$r=r_*:T=T_*,\quad \lambda_1\frac{\partial T}{\partial r}\Big|_{r=r_*-0}=\lambda_2\frac{\partial T}{\partial r}\Big|_{r=r_*+0}-\rho_1 Q_*\frac{\partial r_*}{\partial t_*},\quad v=0; \quad (1.4)$$

$$r=0:\frac{\partial T}{\partial r}=0,\quad \frac{\partial v}{\partial r}=0,\quad r=r_1:T=T_0, \quad (1.5)$$

where r is the present radius; r_1 is the pipe's radius; v is the velocity of the liquid; Q_* is the heat of the phase transition; a_1, a_2, λ_1, and λ_2 are the coefficients of thermal diffusivity and thermal conductivity for the liquid and the solid phase, respectively; ρ_1 is the liquid density; τ_* is the shearing stress; and $b = -\partial p/\partial z$ is the pressure gradient along the length of the tube, which from now on will be considered constant.

In dimensionless variables

$$\Theta=\frac{E}{RT_*^2}(T-T_*),\quad x=\frac{r}{r_1},\quad \omega=\frac{\eta(T_*)v}{r_1^2 b},\quad \tau=\frac{RT_*^2}{E}\frac{\lambda_1}{Q_*\rho_1 r_1^2}t,\quad x_*=\frac{r_*}{r_1}$$

and in the exponential curve approximation of Frank-Kamenetskii [10] ($RT_*/E \ll 1$), Eqs. (1.1)-(1.3) and boundary conditions (1.4) and (1.5) take the form

$$x<x_*:\varepsilon\frac{\partial\Theta}{\partial\tau}=\frac{\partial^2\Theta}{\partial x^2}+\frac{1}{x}\frac{\partial\Theta}{\partial x}+16\varkappa\exp(-\Theta)\left(\frac{\partial\omega}{\partial x}\right)^2; \quad (1.6)$$

$$\varepsilon_1\frac{\partial\omega}{\partial\tau}=\frac{1}{x}\frac{\partial}{\partial x}\left(\exp(-\Theta)\cdot x\frac{\partial\omega}{\partial x}\right)+1; \quad (1.7)$$

$$x>x_*:\frac{\varepsilon}{a}\frac{\partial\Theta}{\partial\tau}=\frac{\partial^2\Theta}{\partial x^2}+\frac{1}{x}\frac{\partial\Theta}{\partial x}; \quad (1.8)$$

$$x=x_*:\lambda\frac{\partial\Theta}{\partial x}\Big|_{x=x_*+0}=\frac{\partial\Theta}{\partial x}\Big|_{x=x_*-0}+\frac{\partial x_*}{\partial\tau},\quad \Theta=0,\quad \omega=0; \quad (1.9)$$

$$x=0:\frac{\partial\Theta}{\partial x}=0,\quad \frac{\partial\omega}{\partial x}=0,\quad x=1:\Theta=\Theta_0. \quad (1.10)$$

The parameters λ, κ, Θ_0, a, ε, ε_1, and the definition of s given below are determined by equations

$$\lambda=\frac{\lambda_2}{\lambda_1},\quad \varkappa=\frac{Er_1^4 b^2}{16RT_*^2\lambda_1\eta(T_*)},\quad \Theta_0=\frac{E}{RT_*^2}(T_0-T_*),$$
$$a=\frac{a_2}{a_1},\quad \varepsilon=\frac{\lambda_1}{a_1\rho_1 Q_*}\frac{RT_*^2}{E},\quad \varepsilon_1=\frac{\lambda_1}{\eta(T_*)Q_*}\frac{RT_*^2}{E},\quad s=-\frac{\lambda\Theta_0}{2}. \quad (1.11)$$

We will first consider the steady-state problem. For solving the steady-state equations (1.6)-(1.8), it is useful to make the substitution of variables suggested in [11, 12]: $\xi = x^2$. Excluding the velocity gradient, Eqs. (1.6)-(1.8) and boundary conditions (1.9) and (1.10) then take the form

$$\xi<\xi_*:\frac{d^2\Theta}{d\xi^2}+\frac{1}{\xi}\frac{d\Theta}{d\xi}+\varkappa\exp\Theta=0; \quad (1.12)$$

$$\xi>\xi_*:\frac{d^2\Theta}{d\xi^2}+\frac{1}{\xi}\frac{d\Theta}{d\xi}=0; \quad (1.13)$$

$$\xi=\xi_*:\Theta=0,\quad \Theta'_-=\lambda\Theta'_+; \quad (1.14)$$

$$\xi = 0: d\Theta/d\xi = 0,\ \xi = 1: \Theta = \Theta_0, \tag{1.15}$$

where Θ'_- and Θ'_+ are the maximum values of $d\Theta/d\xi$ while moving from the left and from the right to the point $\xi = \xi_*$ ($\xi_* = x^2_*$).

The solution of Eqs. (1.11) and (1.12) using boundary conditions (1.13) and (1.14) can be found in [7]:

$$\xi_* < \xi < 1: \Theta = \Theta_0 (1 - \ln \xi / \ln \xi_*); \tag{1.16}$$

$$0 < \xi < \xi_*: \Theta = 2 \ln \left[\frac{4}{4 - q(1 - \xi^2/\xi_*^2)} \right], \quad \varkappa \xi_*^2 = q(4 - q)/2; \tag{1.17}$$

$$\Theta'_- = -q/\xi_*, \quad \Theta'_+ = -\Theta_0/\xi_* \ln \xi_*, \quad q = -2s/\ln \xi_*. \tag{1.18}$$

Equations (1.16)-(1.18) determine the solution of the problem, where q can be excluded according to (1.18).

2. Flow Conditions with a Given Pressure Gradient. For liquid flow with a given pressure gradient, one can assume three ways for the development of the process: the well-known case of a capped pipe (total phase transformation), a flow with intermediate positioning of the phase boundary, and hydrodynamic thermal shock (HTS). The effect of a loss of the steady-state solution to the problem for sufficiently large pressure gradients is labeled as such using the terminology in [9]. In [13], attention was further given to the idealization of the theoretical analysis in [9]. However, as was indicated in [13], the model of HTS in [9] is of interest as a limiting case which allows one to consider the critical conditions of hydrodynamic ignition for a tube of sufficiently long dimensions (the transition from low temperature with a small flow rate to high temperature with a large flow rate).

We will notice that the problem we are considering here becomes an HTS problem [9] for a wall temperature T_0 equal to the phase transformation temperature T_*, i.e., $\Theta_0 = 0$. The limiting case $\kappa \to \infty$ (sufficiently high heat release) evidently corresponds to shock conditions. In another limiting case $\Theta_0 \to -\infty$ (for a high degree of cooling), conditions of capping exist.

An analysis of solutions (1.16)-(1.18) to Eqs. (1.12)-(1.15), which describe the steady-state temperature distribution of the liquid for a flow with a given pressure gradient, can be found in [7] with regard to thermal shock under conditions of phase transition. We will here present some of the results from [7].

Substituting q from (1.18) into the last expression of (1.17), one can obtain the dependence of the parameter κ on the coordinate of the phase front ξ_*:

$$\varkappa = 2(-s/\ln \xi_*)(2 + s/\ln \xi_*)/\xi_*^2. \tag{2.1}$$

The last expression in (1.17) gives two values for q:

$$q_\pm = 2\left(1 \pm \sqrt{1 - \varkappa \xi_*^2/2}\right), \tag{2.2}$$

which correspond to the two temperature distributions $\Theta_\pm$ determined by the first equation of (1.17). As is known from thermal shock theory [12, 14], only the distribution Θ_- is stable for relatively small temperature changes.

The value of κ in (2.1) must be positive. This determines the range over which ξ_* can change: $0 < \xi_* < \exp(-s/2)$. It is evident from Eq. (2.1) that $\kappa \to \infty$ as $\xi_* \to 0$ and $\kappa \to 0$ as $\xi_* \to \exp(-s/2)$; consequently, the problem has a solution for any $\kappa > 0$, $s > 0$. Analysis of (2.1) shows [7] that for $0 < s < 3 - 2\sqrt{2} \approx 0.172$ the dependence $\kappa(\xi_*)$ is nonmonotonic and, therefore, in some range for the values of the parameter κ, the solution is not unique. The boundaries on the region of nonmonotonicity of $\kappa_\pm$ are determined by the following relations (Fig. 1):

$$\varkappa_\pm(s) = 2f_\mp(2 - f_\mp) \exp f_\pm; \tag{2.3}$$

$$\xi_\pm = \exp(-f_\pm/2); \tag{2.4}$$

$$f_\pm = (1 + s \pm \sqrt{(1 - s)^2 - 4s})/2. \tag{2.5}$$

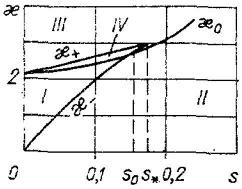

Fig. 1

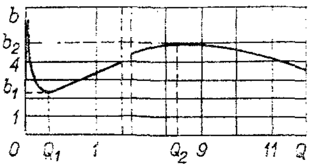

Fig. 2

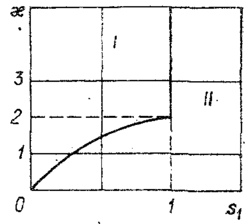

Fig. 3

For $s = s_* = 3 - 2\sqrt{2}$, the values of κ_+ and κ_- merge, and for $s > s_*$, the solution becomes unique. In the nonmonotonic region, the dependence $\kappa(\xi_*)$ decreases for $\xi_* < \xi_+$ and $\xi_* > \xi_-$ and increases for $\xi_+ < \xi_* < \xi_-$.

Because of the physical meaning of ξ_* (ξ_* must increase with an increase in the intensity of heat release, i.e., κ), one can assume that only the increasing branch for the dependence of $\kappa(\xi_*)$ is stable.

An analysis of the equations using the quasisteady-state approximation, when the times for thermal and hydrodynamic relaxation are substantially less than the characteristic time for phase transformation (see [7]), i.e., they fulfill the conditions $\varepsilon \ll 1$, $\varepsilon_1 \ll 1$ [see (1.11)], yields a presentation of the processes for unstable steady states. In this case, the equation of motion for the boundary of the phase front, in dimensionless variables, takes the form

$$\partial\xi_*/\partial\tau = 8\left(s/\ln\xi_* + 1 - \sqrt{1 - \varkappa\xi_*^2/2}\right), \tag{2.6}$$

and the temperature distribution is determined by Eqs. (1.16)-(1.18), taking into account only the lesser (stable) distribution obtained from (1.17) while excluding q. Although Eq. (2.6) differs somewhat from the corresponding equation in [7], it allows for sufficiently complete qualitative research. In particular, the range of possible steady states narrows due to the rejection of those steady-state profiles which are unstable relative to temperature changes. In the plane of the parameters κ and s, the range of steady states for the quasisteady-state problem are determined by the inequality $\kappa < \kappa_0 = 2 \exp(2s)$ (see Fig. 1).

Combining the results from an analysis of the steady-state and quasisteady-state system of equations, we can draw some conclusions.

1. In contrast to [9], the present problem [see (1.6)-(1.10)] has a steady-state solution for any value of the parameter κ. In the range $s < s_*$, $\kappa_-(s) < \kappa < \kappa_+(s)$, the solutions corresponding to $\xi_+(s) < \xi_* < \xi_-(s)$ are stable. Within this range, dependence (2.1) is increasing. The decreasing sections of curve (2.1) correspond to unstable solutions. Within the range of uniquess [a decreasing dependence for (2.1)], there are no stable steady states.

2. The nonsteady-state peculiarities of the flow are determined by the unstable steady states. If instability is exhibited only relative to changes in the phase boundary, but remains stable relative to changes in temperature and velocity, then the above formulated quasisteady-state approximation is permissible.

3. In the quasisteady-state approximation, one can plot the ranges for specific types of flow (see Fig. 1): I is the stable steady-state condition with intermediate positioning of the phase boundary $\xi_+(s) < \xi_* < \xi_-(s)$, II is for tube capping (total phase transformation), III is for hydrodynamic thermal shock, and IV is the section which is common for regions I and III. The stable steady states exist here, but the initial conditions $\xi_* = 1$ for $\tau = 0$ do not pertain to the region of their attraction and result in HTS.

4. The steady states with unstable temperature profiles do not effect the quasisteady-state behavior of the solutions. They are not steady state points for the quasisteady-state problem.

3. Flow Rate and Pressure Characteristics for Flow with a Given Flow Rate. We will consider flow rate and pressure characteristics for flow during phase transition, i.e., the dependence of pressure gradient $b = -\partial p/\partial z$ on the flow rate Q.

Integrating the second equation of (1.7) and taking into account (1.17), we obtain

$$\omega = \frac{1}{\varkappa\xi_*}\left\{\frac{q}{4} - \frac{q\xi\xi_*}{q(\xi^2-\xi_*^2)+4\xi_*^2} + \sqrt{\frac{q}{4-q}}\left[\operatorname{arctg}\sqrt{\frac{q}{4-q}} - \operatorname{arctg}\left(\frac{\xi}{\xi_*}\sqrt{\frac{q}{4-q}}\right)\right]\right\}; \quad (3.1)$$

$$Q = \frac{\pi r_1^2}{\sqrt{\varkappa}}\sqrt{\frac{RT_*^2\lambda_1}{\eta(T_*)E}}\,q. \quad (3.2)$$

Equations (3.2), (2.1), and (2.2) give the parametric requirements of the flow rate for the case of a given temperature on the pipe wall (Fig. 2). During phase transition, this characteristic differs qualitatively from the known curve for nonisothermal flow [15]. The phase transition leads to the appearance of a new decreasing branch for which $b \to \infty$ for $Q \to 0$. Such an abnormal dependence is due to the small amount of dissipative heat release for low flow rates and, consequently, a thick layer of solid substance "grows" on the wall of the pipe, which impedes the flow of liquid ($Q < Q_1$, Fig. 2). For large flow rates ($Q > Q_2$), the resistance to flow decreases due to the great amount of heating in the liquid and to the strong temperature dependence of viscosity (compare with [15]). The dependence b(Q) increases only for some intermediate values of the flow rate ($Q_1 < Q < Q_2$). This branch corresponds to stable steady states for flow with a given pressure gradient. For a given pressure gradient which is less than b_1, capping of the pipe occurs, and for $b > b_2$, HTS develops (see Sec. 2). The steady-state processes for $s > s_*$ are also described in the quasisteady-state approximation, where the flow rate and pressure characteristics begin to decrease monotonely.

The meaning behind the dependence b(Q) allows us to draw some conclusions regarding flow with a given flow rate. We will introduce the dimensionless flow rate

$$\gamma = \frac{Q}{4\pi r_1^2}\sqrt{\frac{E\eta(T_*)}{RT_*^2\lambda_1}}. \quad (3.3)$$

The solution of the problem is determined by (1.16), (1.17), (2.1), and (2.2), where κ is now an unknown quantity which depends on γ and is found from the equation $\sqrt{\kappa}\gamma = -s/2 \ln \xi_*$.

For a given flow rate, both temperature distributions $\Theta_\pm$ [see (1.7) and (2.2)] are stable (compared with [16]), where the plus sign (2.2) should be used for $\gamma > \exp(-s)/\sqrt{8}$ and the minus sign should be used for $\gamma < \exp(-s)/\sqrt{8}$. The dependence of the position of the phase front on the flow rate is obtained from the relation

$$\gamma = [(-s/8\ln\xi_*)/(2 - s/\ln\xi_*)]^{1/2}\xi_*.$$

The dependence $\gamma(\xi_*)$ in the interval $0 < \xi_* < \exp(-s/2)$ is a monotonely increasing function, i.e., for any given flow rate, there exists a unique steady-state position of the phase front ξ_*, where ξ_* can be take on any value in the interval $0 < \xi_* < \exp(-s/2)$.

We note that the thickness of the "frozen" layer decreases monotonely with an increase in the flow rate, and the layers of solid substance on the pipe wall are substantial for arbitrarily large flow rates.

4. The Case of a Given Thermal Flow on the Pipe Wall. For a given thermal flow on the pipe wall ($r = r_1$: $\partial T/\partial r = \bar{q}_0$ or, in dimensionless variables, $\xi = 1$: $\partial\Theta/\partial\xi = q_0$, $q_0 = \bar{q}_0 r_1 E/2RT_*^2$), we will write the solution of the system of equations (1.11)-(1.14) in the form [compare with [7] and (1.16)-(1.18)]

$$\xi_* < \xi < 1 : \Theta = -q_0 \ln(\xi_*/\xi); \quad (4.1)$$

$$0 < \xi < \xi_* : \Theta = 2\ln\left[\frac{2}{2 - s_1(1-\xi^2/\xi_*^2)}\right], \quad (4.2)$$

$$\varkappa\xi_*^2 = 2s_1(2 - s_1), \quad s_1 = -\frac{q_0\lambda}{2}.$$

The dependence $\kappa(\xi_*)$ then decreases monotonely, and the solution is unstable for all ranges of the parameters. The quasisteady-state equation of motion for the boundary of the phase division (1.9) has the form

$$\partial\xi_*/\partial\tau = 8\left(1 - s_1 - \sqrt{1 - \varkappa\xi_*^2/2}\right). \quad (4.3)$$

In Fig. 3, the boundary which divides region I (conditions of HTS) and region II (conditions of pipe capping) is determined by relations

$$s_1 = 1 - \sqrt{1 - \varkappa/2} \text{ for } \varkappa < 2 \text{ and } s_1 = 1 \text{ for } \varkappa \geqslant 2. \tag{4.4}$$

Upon integrating Eq. (4.3), one can obtain the time of complete pipe capping for $\kappa < 2$:

$$0 < s_1 < 2: \tau_{*1} = \frac{1}{8}\sqrt{\frac{2}{\varkappa}}\left[Y - \frac{(1-s_1)}{\sqrt{s_1(2-s_1)}} \ln\left|\frac{(2-s_1)\,\mathrm{tg}(Y/2) - \sqrt{s_1(2-s_1)}}{(2-s_1)\,\mathrm{tg}(Y/2) + \sqrt{s_1(2-s_1)}}\right|\right],$$
$$s_1 > 2: \tau_{*1} = \frac{1}{8}\sqrt{\frac{2}{\varkappa}}\left\{Y - \frac{2(s_1-1)}{\sqrt{s_1(s_1-2)}} \operatorname{arctg}\left[\frac{(s_1-2)\,\mathrm{tg}(Y/2)}{\sqrt{s_1(s_1-2)}}\right]\right\}, \tag{4.5}$$
$$Y = \arcsin\sqrt{\varkappa/2}.$$

For $\kappa > 2$, the process of pipe capping can be divided into two stages: establishment of the quasisteady-state temperature profiles for $\xi_* > \xi_0$ ($\xi_0 = \sqrt{2/\kappa}$) and passage of the phase front, under quasisteady-state conditions, from ξ_0 to 0. The time for the first stage cannot be found in the quasisteady-state approximation, but one can determine the time for the second stage τ_{*2}. The total time of pipe capping will always be greater than τ_{*2}:

$$0 < s_1 < 2: \tau_{*2} = \frac{1}{8}\sqrt{\frac{2}{\varkappa}}\left[\frac{\pi}{2} - \frac{(1-s_1)}{\sqrt{s_1(2-s_1)}} \ln\left|\frac{(2-s_1) - \sqrt{s_1(2-s_1)}}{(2-s_1) + \sqrt{s_1(2-s_1)}}\right|\right],$$
$$s_1 > 2: \tau_{*2} = \frac{1}{8}\sqrt{\frac{2}{\varkappa}}\left\{\frac{\pi}{2} + \frac{2(1-s_1)}{\sqrt{s_1(s_1-2)}} \operatorname{arctg}\left[\frac{(s_1-2)}{\sqrt{s_1(s_1-2)}}\right]\right\}. \tag{4.6}$$

Expressions (4.5) and (4.6) apply only in region II (see Fig. 3). Hydrodynamic thermal shock develops in region I. The development of HTS can also be divided into two stages: establishment of the quasisteady-state velocity and temperature profiles and development of HTS under quasisteady-state conditions. During the first stage, a layer of solid substance is able to form on the walls of the pipe. During the second stage, the temperature of the liquid increases, and the layer of solid substance begins to decrease. The time for the second stage is determined from (4.3)

$$\tau_{\text{shock}} = \frac{1}{8}\sqrt{\frac{2}{\varkappa}}\left\{Y_1 - \frac{\pi}{2} + \frac{(1-s_1)}{\sqrt{s_1(2-s_1)}} \ln\left|\frac{\left[(2-s_1) - \sqrt{s_1(2-s_1)}\right]\left[(2-s_1)\,\mathrm{tg}\frac{Y_1}{2} + \sqrt{s_1(2-s_1)}\right]}{\left[(2-s_1) + \sqrt{s_1(2-s_1)}\right]\left[(2-s_1)\,\mathrm{tg}\frac{Y_1}{2} - \sqrt{s_1(2-s_1)}\right]}\right|\right\}. \tag{4.7}$$

Here $Y_1 = \arcsin(\sqrt{\kappa/2}\xi_{*0})$; ξ_{*0} is the minimum coordinate of the phase front which is attained during the first stage of nonsteady-state development of HTS. This quantity cannot be determined in the quasisteady-state approximation and, therefore, Eq. (4.7), in contrast to (4.5), does not allow for the calculation of τ_{shock}. One can, however, make a qualitative conclusion about the limits of applicability for the quasisteady-state approximation; $\tau_{\text{shock}} = 0$ for $\kappa\xi^2_{*0}/2 = 1$, and for $\kappa\xi^2_{*0}/2 > 1$, the quantity Y_1 and, consequently τ_{shock}, lose their meaning. The quasisteady-state approximation is not possible in this range; thermal shock occurs during the first stage of the nonsteady-state development of the process. This range depends, generally speaking, on all of the parameters of the problem and, in particular, on the heat of the phase transition Q_*. The range, therefore, corresponds to large Q_*. The quasisteady-state approximation is evidently valid for intermediate values of Q_* which are not too large, so as to fulfill the condition $\kappa\xi^2_*/2 < 1$, and not too small, so as to fulfill the inequality $\varepsilon \ll 1$ and $\varepsilon_1 \ll 1$ [see (1.1)].

For a given flow rate, the solution of the problem is given by relations (4.1)-(4.3). The unknown quantity κ is determined by the relation $\kappa = s_1^2/4\gamma$, where γ is the dimensionless flow rate of the liquid (3.3). For boundary conditions of the second kind, one can find the flow rate and pressure characteristics explicitly: $\partial p/\partial z = 2\pi\lambda_2 r_1 q_0/Q$.

In contrast to the isothermal flow of Newtonian liquid, where the pressure gradient is directly proportional to the flow rate, the pressure gradient in the present problem is inversely proportional to the flow rate, which explains the accompanying effects of dissipative heating and phase transition.

The dependence of γ on ξ_* monotonely increases: $\gamma = \xi_*[s_1/8(2 - s_1)]^{1/2}$, where, as a result of intensive heat release, there will only be liquid in the pipe. In the plane of the parameters γ, s_1, the boundary which divides the regions of flow with one and two phases has the form $\gamma = \sqrt{s_1/8(2 - s_1)}$.

LITERATURE CITED

1. S. S. Grigoryan, "Heating and melting of a solid object due to friction," Prikl. Mat. Mekh., 22, No. 5 (1958).
2. V. M. Agapkin, B. A. Krivoshein, and V. A. Yufin, Thermal and Hydraulic Consideration of Pipelines for Petroleum and Petroleum Products [in Russian], Nedra, Moscow (1981).
3. S. A. Fedotov, "The rise of basal magma from the earth's core and the mechanism of basaltic fissure eruptions," Izv. Akad. Nauk SSSR, Ser. Geol., No. 10 (1976).
4. M. N. Ozisik and J. C. Mulligan, "Transient freezing of liquids in forced flow inside circular tubes," Trans. ASME, J. Heat Transfer, 91, No. 3 (1969).
5. V. N. Dorovskii, A. M. Iskol'dskii, and E. I. Romenskii, "Modeling the hydrodynamic stage of phase transformations for electrical shock of metals," Preprint No. 83-91, Inst. Yad. Fiz., Sib. Otd. Akad. Nauk SSSR (1983).
6. S. I. Khudyaev, A. M. Stolin, and S. V. Maklakov, "Thermal shock under conditions of phase transformation," FGV, No. 5 (1983).
7. S. I. Khudyaev and A. M. Stolin, "Analysis of the conditions of self-ignition in cylindrical volumes for frontal phase transformation," Khim. Fiz., No. 9 (1984).
8. S. V. Maklakov, A. M. Stolin, and S. I. Khudyaev, "Phase transition under conditions of nonisothermal Couette flow of liquid," Zh. Prikl. Mekh. Tekh. Fiz., No. 4 (1984).
9. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, "Hydrodynamic thermal 'shock'," Dokl. Akad. Nauk SSSR, 163, No. 1 (1965).
10. D. A. Frank-Kamenetskii, Diffusion and Thermal Transmission in Chemical Kinetics [in Russian], Nauka, Moscow (1967).
11. S. I. Khudyaev, "Some estimations of the eigenvalues for spherically symmetric problems," in: Methods for Solving Problems in Mathematical Physics [in Russian], Nauka, Moscow (1966).
12. A.I. Vol'pert and S.I. Khudyaev, Analysis of Discontinuous Functions and Equations in Mathematical Physics [in Russian], Nauka, Moscow (1975).
13. A. G. Merzhanov and A. M. Stolin, "Hydrodynamic analogs for the phenomena of ignition and extinction," Zh. Prikl. Mekh. Tekh. Fiz., No. 1 (1974).
14. A. G. Istratov and V. B. Librovich, "The stability of solutions in the steady-state theory of thermal shock," Prikl. Mat. Mekh., 27, No. 2 (1963).
15. A. M. Stolin, "Nonisothermal flow of liquid in a capillary. Nonisothermal flow of liquid in a rotary viscosimeter," in: Diffusion and Viscosity of Polymers. Methods of Measurement [in Russian], Khimiya, Moscow (1979).
16. S. A. Bostandzhiyan, A. G. Merzhanov, and S. I. Khudyaev, "Several problems for nonisothermal steady-state flow of viscous liquid," Zh. Prikl. Mekh. Tekh. Fiz., No. 5 (1965).